

DPPS

$$a+b=6 \rightarrow \frac{a}{a} + \frac{1}{b}$$

$$\frac{\frac{a}{2} + \frac{a}{2} + b}{3} \geq \frac{3}{\frac{2}{a} + \frac{2}{a} + b}$$

$$(2) HM = \frac{10}{\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{10}}} = \frac{\lambda}{2^{10}-1}$$

$$= \frac{2^{10} \times 10}{2^9 + 2^8 + 2^7 + \dots + 2^0}$$

← GP →

Q3 $\Psi \Omega$

$$\left. \begin{array}{l} AM = x^2 + 1 \\ GM = 2x + 1 \\ HM = 4 \end{array} \right\} L^2 = AH$$

Q1 $x^3 - 6x^2 + 3px - 2p = 0$

↖ α
↗ β
↘ γ

$$\alpha + \beta + \gamma = 6$$

$$\overline{\alpha\beta} = 3p$$

$$\alpha \cdot \beta \cdot \gamma = 2p$$

$$AM = \frac{6}{3} = 2$$

$$HM = \frac{3}{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}$$

$$= \frac{3\alpha\beta\gamma}{\alpha\beta + \beta\gamma + \gamma\alpha} = \frac{3 \times 2p}{3p} = 2$$

$$AM = HM \Rightarrow \alpha = \beta = \gamma = 2$$

Q 5

$$a+b+c=3 \Rightarrow 6-x+6-y+6-z=3$$

$$\left(\frac{a}{6-a} + \frac{b}{6-b} + \frac{c}{6-c} \right) = ? \quad \text{Min} \quad \Rightarrow x+y+z=15$$

$$6-a=x, 6-b=y, 6-c=z$$

$$a=6-x \quad b=6-y \quad c=6-z$$

$$\frac{6-x}{x} + \frac{6-y}{y} + \frac{6-z}{z}$$

$$\left(6 \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) - 3 \right)_{\text{Min}} = 6 \left(\frac{3}{5} \right) - 3 = \frac{3}{5}$$

$$\frac{x+y+z}{3} \geq \frac{3}{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)} \Rightarrow \frac{5}{3} \geq \frac{3}{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)} \Rightarrow \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \geq \left(\frac{3}{5} \right)$$

$$A_4 H_7 = \frac{26}{11} \times \frac{66}{26}$$

$$Q8 \quad f(x) = (a^2 + b^2 - 4a - 6b + 13)(2x^2 - 4x + 5)$$

$$f(0) = f(1) = f(2) \quad d = \frac{\frac{1}{3} - \frac{1}{2}}{11} = -\frac{1}{66}$$

$$\frac{1}{10} \sum_{r=4}^8 A_r H_{11-r} = \frac{1}{10} (A_4 H_7 + A_5 H_6 + A_6 H_5 + A_7 H_4 + A_8 H_3) = \frac{3}{10} \times \frac{7}{2} = 3$$

$$\begin{cases} 5(a^2 + b^2 - 4a - 6b + 13) = 3(a^2 + b^2 - 4a - 6b + 13) \\ 5(a-2)^2 + (b-3)^2 = 3(a-2)^2 + (b-3)^2 \end{cases}$$

$$a=2, b=3$$

$$2, A_1, A_2, \dots, A_{10}, 3 \quad d = \frac{3-2}{11} = \frac{1}{11}$$

$$A_4 = 2 + 4d = 2 + \frac{4}{11} = \frac{26}{11}$$

$$H_7 = \frac{1}{2} + 7d = \frac{1}{2} - \frac{7}{66} = \frac{33-7}{66} = \frac{26}{66}$$

$$\begin{array}{l}
 P(x) = 5x^4 + p x^3 + q x^2 + r x + s \\
 \left. \begin{array}{l} \alpha \\ \beta \\ \gamma \\ \delta \end{array} \right\} \text{B.R.} \quad \bigg| \quad Q(x) = x^3 - \boxed{9} x^2 + a x - 24 \quad \left. \begin{array}{l} \alpha \\ \gamma \\ \delta \end{array} \right\} \text{A.P.}
 \end{array}$$

$$\begin{array}{l}
 \alpha + \gamma + \delta = 9 \quad \bigg| \quad \alpha \cdot \gamma \cdot \delta = 24 \\
 3\gamma = 9 \quad \bigg| \quad \alpha \cdot \delta = 8 \\
 \boxed{\gamma = 3} \quad \alpha + \delta = 6 \\
 \underline{\alpha = 4, \delta = 2}
 \end{array}$$

$$2\gamma = \alpha + \delta$$

$$A+B+C=\pi \text{ (give)} \Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \Rightarrow \boxed{\frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right)}$$

$$Q \quad \cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$$

$$\stackrel{LHS}{=} 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) + 2 \cos^2 \frac{C}{2} - 1$$

$$2 \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right) + 1 - 2 \sin^2 \frac{C}{2}$$

$$\underline{2 \sin \frac{C}{2} \cdot \cos \left(\frac{A-B}{2} \right) + 1 - 2 \sin^2 \frac{C}{2}}$$

$$1 + 2 \sin \frac{C}{2} \left\{ \cos \left(\frac{A-B}{2} \right) - \sin \left(\frac{\pi}{2} - \frac{A+B}{2} \right) \right\}$$

$$1 + 2 \sin \frac{C}{2} \left\{ \cos \left(\frac{A-B}{2} \right) - \cos \left(\frac{A+B}{2} \right) \right\} \stackrel{\cos C - \cos D}{=} -2 \sin \frac{C}{2} \sin \frac{A+B}{2}$$

$$1 + 2 \sin \frac{C}{2} \left\{ + 2 \sin \left(\frac{A}{2} \right) \cdot \sin \left(\frac{B}{2} \right) \right\} = RHS$$

$$A+B+C=\pi \quad B+C=\pi-A$$

$$Q \quad \sin \left(\frac{B+C-A}{\pi-A} \right) + \sin \left(\frac{C+A-B}{\pi-B} \right) + \sin \left(\frac{A+B-C}{\pi-C} \right) = ?$$

$$A+B+C=\pi$$

$$B+C=\pi-A$$

$$\sin(\pi-2A) + \sin(\pi-2B) + \sin(\pi-2C)$$

$$\sin 2A + \sin 2B + \sin 2C$$

$$= 4 \sin A \cdot \sin B \cdot \sin C$$

$$\sin^2 C - \sin^2 B = \sin(B+C) \sin(B-C)$$

$$\sin(\pi-A)$$

$$Q. \quad \cos^2 A + \cos^2 B - \cos^2 C = ?$$

$$\cos^2 A + \frac{\sin^2 C - \sin^2 B}{\pi-A}$$

$$1 - \sin^2 A + \sin(B+C) \sin(B-C)$$

$$1 - \sin^2 A + \sin A \cdot \sin(B-C)$$

$$1 - \sin A \{ \sin A + \sin(B-C) \}$$

$$\pi - (B+C)$$

$$1 - \sin A \{ \sin(B+C) + \sin(B-C) \}$$

$$1 - \sin A \{ 2 \sin B \cos C \}$$

$$1 - 2 \sin A \cdot \sin B \cdot \sin C$$

Result

$$\tan(4\theta + 2\theta + \theta) = 0$$

$$Q \quad \alpha + \beta = \gamma$$

$$\text{then } \tan^2 \alpha + \tan^2 \beta + \tan^2 \gamma = ?$$

$$\tan^2 \alpha + 1 - \tan^2 \beta + \tan^2 \gamma$$

$$1 + (\tan^2 \alpha - \tan^2 \beta) + \tan^2 (\alpha + \beta)$$

$$1 + \tan(\alpha + \beta) \cdot \tan(\alpha - \beta) + \tan^2(\alpha + \beta)$$

$$1 + \tan(\alpha + \beta) (\tan(\alpha - \beta) + \tan(\alpha + \beta))$$

$$1 + \tan \gamma (2 \tan \alpha \cdot \tan \beta)$$

$$1 + 2 \tan \alpha \cdot \tan \beta \cdot \tan \gamma$$

$$Q \quad 6\theta = 2\pi \quad \text{P.T.} \quad \boxed{7\theta = 2\pi}$$

$$3\theta = 2\pi - 4\theta$$

$$4\theta + 2\theta + \theta = 7\theta = 2\pi$$

$$\{\tan \theta \cdot \tan 2\theta + \tan 2\theta \cdot \tan 4\theta + \tan 4\theta \cdot \tan \theta\} = -1$$

To Prove

$$(-\tan \theta \cdot \tan 2\theta + 1) + (\tan 2\theta \cdot \tan 4\theta + 1) + (\tan 4\theta \cdot \tan \theta + 1) = -1$$

$$\frac{\tan 2\theta - \tan \theta}{\tan \theta} + \frac{\tan 4\theta - \tan 2\theta}{\tan 2\theta} + \frac{\tan 4\theta - \tan \theta}{\tan \theta} = -1$$

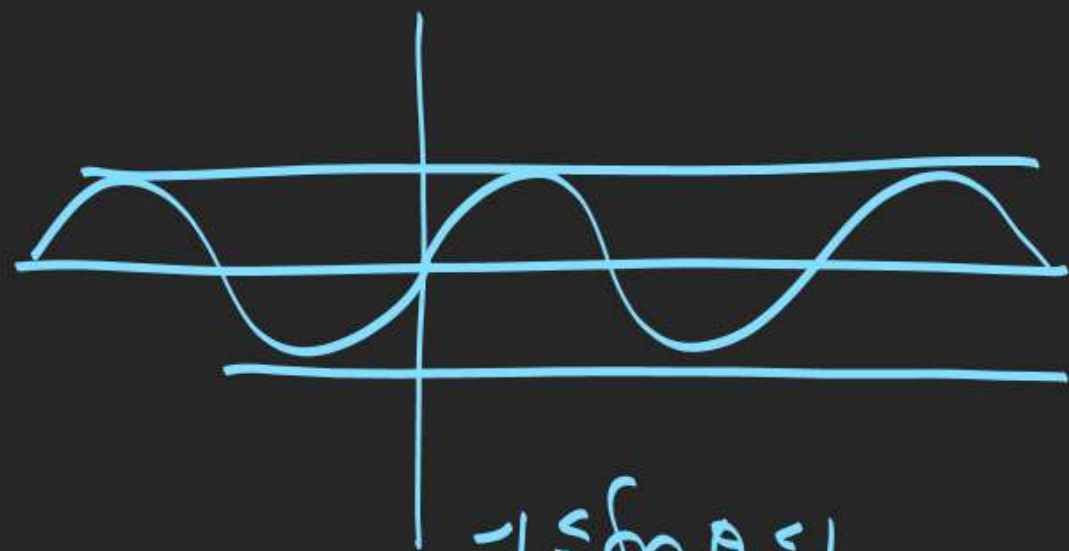
$$\frac{\tan 2\theta}{\tan \theta} - 1 + \frac{\tan 4\theta}{\tan 2\theta} - 1 + \frac{\tan \theta}{\tan 4\theta} - 1 = -1$$

$$\frac{\tan 2\theta}{\tan \theta} + \frac{\tan 4\theta}{\tan 2\theta} + \frac{\tan \theta}{\tan 4\theta} = -1$$

$$\tan(40 + 20 + 0) = \frac{\tan 40 + \tan 20 + \tan 0 - \tan 40 \cdot \tan 20 \cdot \tan 0}{1 - (\tan 40 \cdot \tan 20 + \dots)} = 0$$

$$\tan 40 + \tan 20 + \tan 0 = \tan 40 \cdot \tan 20 \cdot \tan 0$$

① $y = \sin \theta$

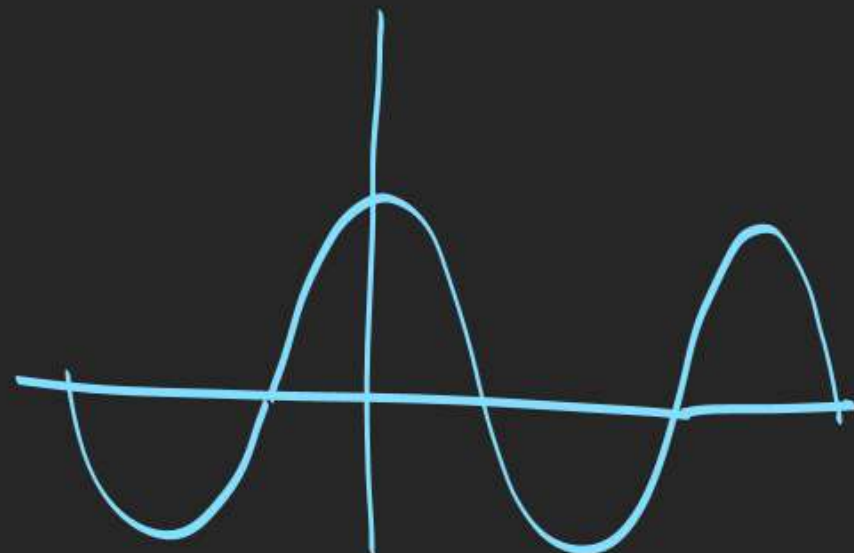


$$-1 \leq \sin \theta \leq 1$$

$$-1 \leq \sin(x) \leq 1$$

$$-1 \leq \sin(ax+b) \leq 1$$

② $y = \cos x$



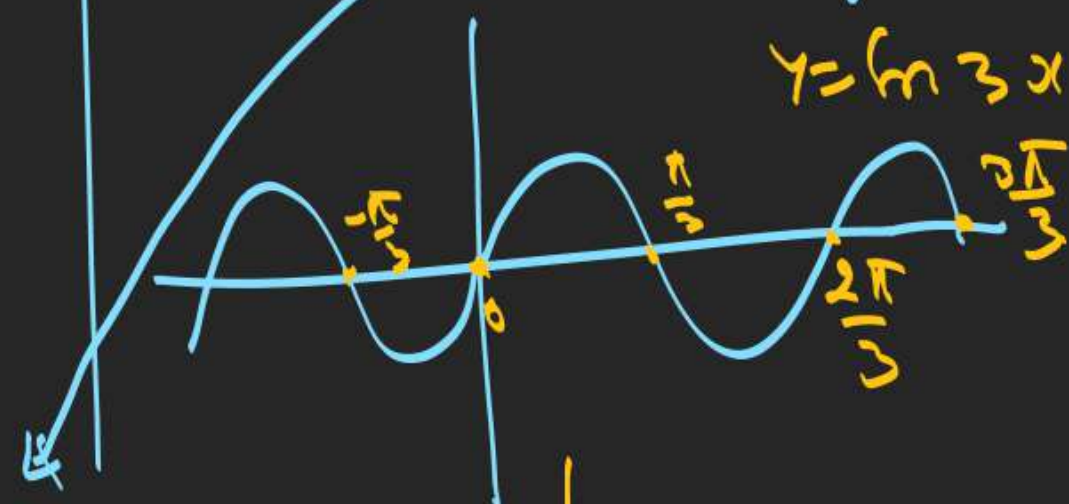
$$-1 \leq \cos x \leq 1$$

$$-1 \leq \cos(ax+b) \leq 1$$

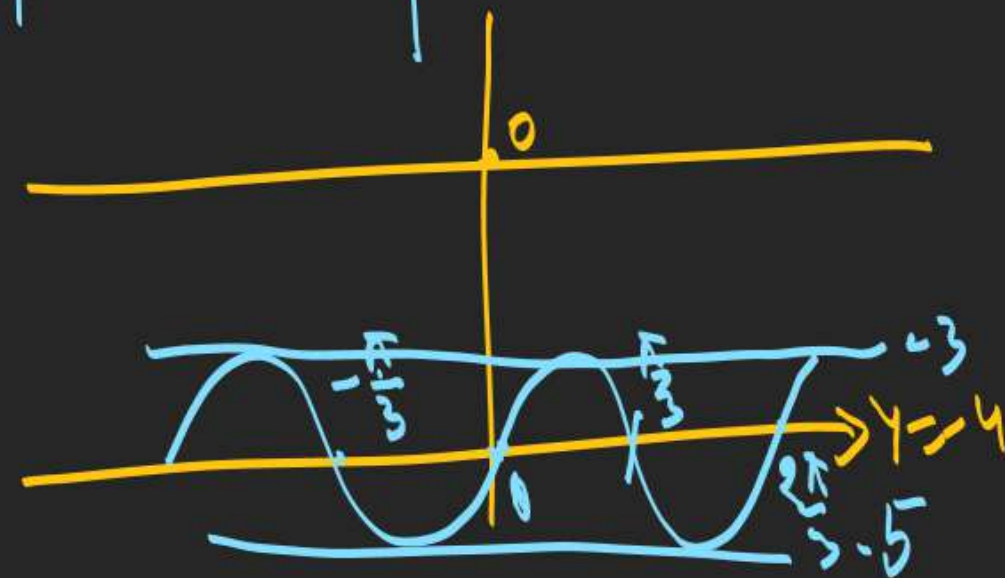
Q $y = \sin(3x-4)$ is R.f?

$\downarrow$
 $[-1, 1]$

Q $y = \sin(3x) - 4$ R.f.



$[-5, -3]$



$$Q \quad y = \sin^2 x \text{ 's } R_f$$

$$-1 \leq \sin x \leq 1$$

$$0 \leq \sin^2 x \leq 1$$

$$R_f \in [0, 1]$$

$$Q \quad y = \cos^2 x \text{ 's } R_f$$

$$\cos x \in [-1, 1]$$

$$\cos^2 x \in [0, 1]$$

$$Q \quad y = \tan x \text{ 's } R_f$$

$$\tan x \in \mathbb{R}$$



$$Q \quad y = \tan^2 x \text{ 's } R_f?$$

$$[0, \infty)$$

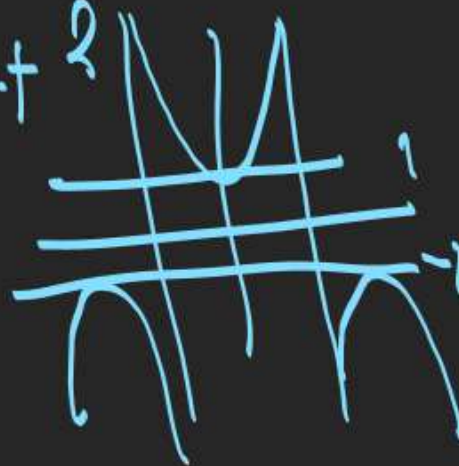
$$Q \quad y = \cot x \text{ 's } R_f$$

$$R_f \in (-\infty, \infty)$$



$$Q \quad y = \sec x \text{ 's } R_f?$$

$$y \in (-\infty, -1] \cup [1, \infty)$$



$$Q \quad y = \sec^2 x \text{ 's } R_f$$

$$\sec^2 x \in [1, \infty)$$

$$Q \quad y = \sec^2 x + 5 \text{ 's } R_f$$

$$\sec^2 x \in [1, \infty)$$

$$\sec^2 x + 5 \in [1+5, \infty)$$

$$\in [6, \infty)$$

Q $y = \frac{\sin x}{\sin x}$ is a graph.

$y = 1$

$R_+ \in \{1\}$

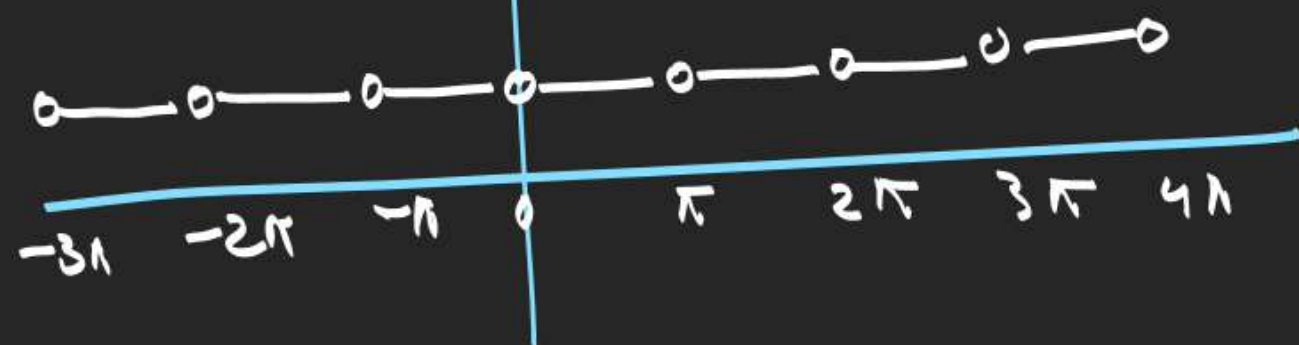
$\sin x$ Dr. Mehar

$\sin x \neq 0$

$x \neq n\pi$

It is a Periodic fn

Period = 1

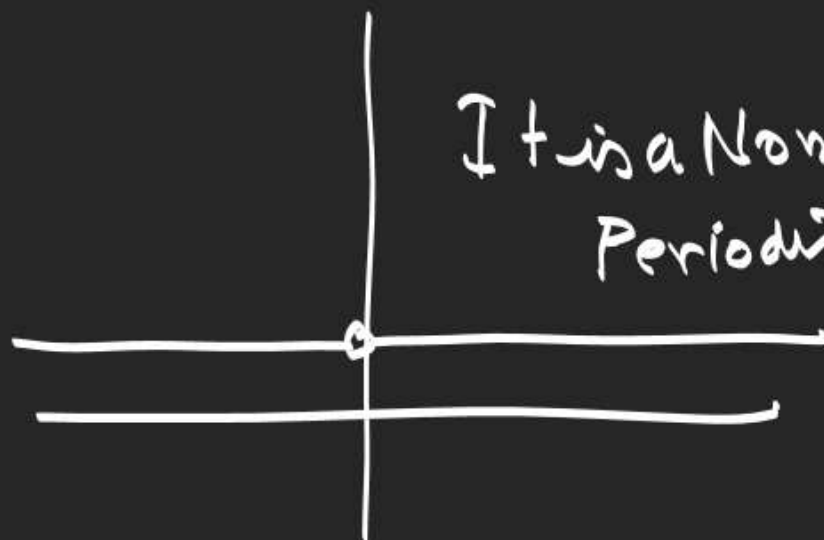


$y = \frac{x}{x}$ is a graph.

$y = 1$

$x \neq 0$

It is a Non Periodic fn



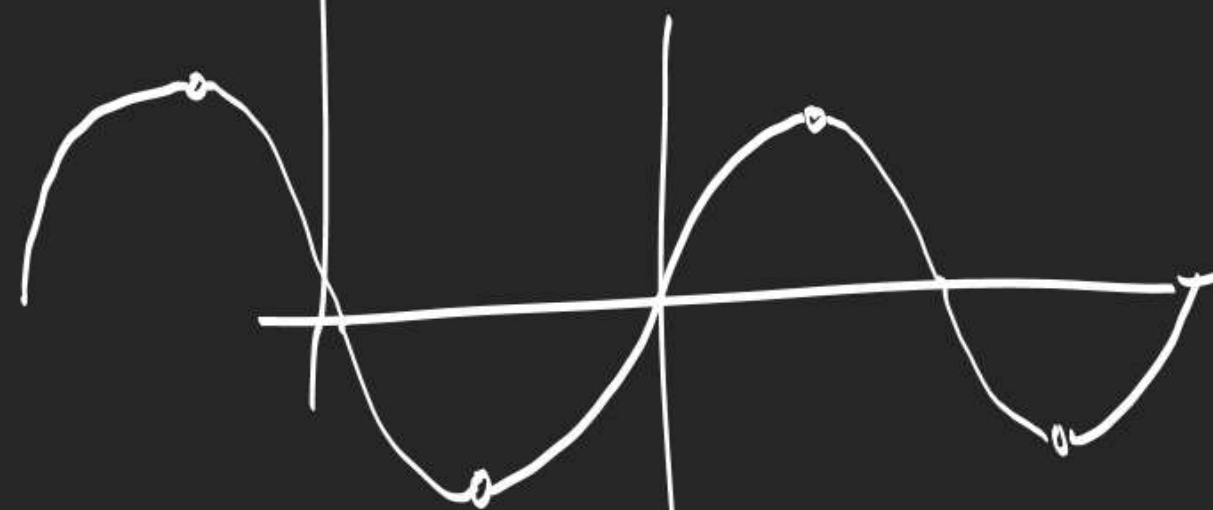
Q $y = 4 \tan x \cdot \cos x$

$= 4 \frac{\sin x}{\cos x} \times \cos x$

$\cos x$ is in Dr.

$\cos x \neq 0$

$x \neq (2n+1)\frac{\pi}{2}$



$\therefore R_+ \in (-1, 1)$

$$Q \ y = \cos^2 \frac{x}{4} - \sin^2 \frac{x}{4}.$$

$$\begin{array}{c} \cos^2 \theta - \sin^2 \theta \text{ Recall.} \\ \text{"} \\ \cos 2\theta \end{array}$$

$$y = \cos\left(\frac{x}{2}\right) \in [-1, 1]$$

$$Q \text{ R+ of } y = \sqrt{\sin x}$$

$$\sqrt{\sin x} \in [0, 1]$$

$$Q \ y = \sin x^2 \text{ R+}$$

$$x = \sqrt{\frac{\pi}{2}} \Rightarrow y = \sin \frac{\pi}{2} = 1$$

$$y = \sqrt{\frac{3\pi}{2}} \Rightarrow y = \sin \frac{3\pi}{2} = -1$$

$$y = \sqrt{\pi} \Rightarrow \sin \pi = 0$$

$$\therefore y = \sin x^2 \in [-1, 1]$$

$$Q \ y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right) \text{ R+}$$

$$\text{Recall } \sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$$

$$\sin\left(\frac{15\pi}{8} - 4x + \frac{17\pi}{8} - 4x\right) \sin\left(\frac{15\pi}{8} - 4x - \frac{17\pi}{8} - 4x\right)$$

$$\sin(4\pi - 8x) \cdot \sin\left(-\frac{2\pi}{8}\right) = +\frac{1}{\sqrt{2}} \boxed{\sin 8x} \in [-1, 1]$$

$$\therefore y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$